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International Journal of Thermal Sciences 43 (2004) 43–51

International Journal of Thermal Sciences
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A generalized procedure for determining thermal resistivity of soils

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Received 12 September 2002; accepted 15 April 2003

Abstract

Estimation of thermal resistivity of soils is very important for various engineering projects. Many researchers have demonstrated that, soil thermal resistivity is a property of the soil that depends on various parameters such as type of soil, particle size distribution, and compaction characteristics and hence its estimation based on existing empirical and mathematical models is difficult. This calls for fabrication of a device that can be used for determining soil thermal resistivity directly. Usually, small size, laboratory thermal resistivity probes have been used for this purpose and their efficiency in measuring soil thermal resistivity has already been established. However, as natural soils consist of various size fractions, ranging from clay to gravel, the laboratory thermal probes cannot be used very efficiently. This necessitates fabrication of a field thermal probe that can be used to measure thermal resistivity of a soil either in its remolded state or under in situ conditions. With this in view, efforts were made to develop a field thermal probe, which works on the principle of transient method and is a magnified version of the laboratory thermal probe developed by the authors. Based on the results obtained efforts have been made to develop generalized relationships for estimating the soil thermal resistivity by knowing the dry density, moisture content and percent size fraction of the various particle sizes, and validation of the proposed generalized equations have been done with the results available in the literature.
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Keywords: Thermal resistivity probes; Clay; Fly ash; Silt; Sand; Gravel

1. Introduction

Estimation of soil thermal resistivity is of great importance for various engineering projects where heat transfer takes place through the soil mass. Some of these projects are; design and laying of high voltage buried power cables [1], oil and gas pipe lines [2], nuclear waste disposal facilities [3], ground modification techniques employing heating and freezing [4] and studies on soil shrinkage [5] etc. These studies indicate that soil thermal resistivity is a property that depends upon various factors, which may be dependent or independent. Efforts have been made by previous researchers to develop relationships to estimate thermal resistivity of soils, and based on their studies these relationships can be classified as: empirical relationships, which are based on data obtained by measurement and analyzed by graphical or numerical techniques [6–16], or theoretical equations, which are based on idealized models wherein the actual soil structure is simplified in such a way to permit mathematical analysis [12].

However, these relationships suffer from limitations in terms of proper incorporation of various factors affecting such a complex phenomenon and to overcome these limitations researchers have used “transient method” of estimation of soil thermal resistivity [17–20]. The method is based on the theory that the rate of rise of the temperature of a line heat source is dependent upon the thermal conductivity of the medium, in which it is placed. The method has been further extended to obtain relationships between various factors affecting the thermal resistivity of the soils and various compositions of the soils have also been tested to simulate a naturally occurring soil deposit [21]. However, these relationships do not incorporate the presence of gravel in the soil mass and its influence on the soil thermal resistivity. This is mainly due to the fact that *a* small, laboratory thermal probe [17,20] cannot be used efficiently to measure thermal resistivity of coarse-grained soils such as gravels and gravely soils.

This calls for fabrication of *a* large, field thermal probe that can be used to measure soil thermal resistivity of a soil sample either remolded to the in situ state of the soil or brought to the laboratory in an undisturbed form or under in situ conditions. However, as the laboratory sample would not represent the actual in situ state of the soil mass, efforts

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doi:10.1016/S1290-0729(03)00103-0

Nomenclature

θ	temperature	K
α	thermal diffusivity constant	$m^2 \cdot s^{-1}$
ξ	resistance per unit length	$\Omega \cdot m^{-1}$
γ_{dry}	dry unit-weight of the soil	$kN \cdot m^{-3}$
γ	unit-weight of the soil	$kN \cdot m^{-3}$
a, b, c	parameters having dependence on type of the soil and moisture content	
C_p	specific heat of the soil	$J \cdot kg^{-1} \cdot K^{-1}$
e_{max}	maximum void ratio	
e_{min}	minimum void ratio	
G_s	specific gravity	
i	current	A
k	thermal conductivity of the soil	$W \cdot K^{-1} \cdot m^{-1}$
L.L.	liquid limit of the soil	%
n	soil porosity	
OMC	optimum moisture content	%
P.L.	plastic limit of the soil	%
Q	heat input per unit length	$J \cdot s^{-1}$
r	radial distance from the heat source	m
R	soil thermal resistivity	$K \cdot m \cdot W^{-1}$
s	slope of straight portion of the temperature versus log(time) relationship	
t	time of heating	s
w	moisture content	%

should be made to measure the soil thermal resistivity with the help of a field thermal probe. In this direction except for [22–24] not many efforts have been made by the researchers.

With this in view, efforts were made to develop a field thermal probe and demonstrate its utility for estimating thermal resistivity of coarse-grained soils such as gravels and gravely soils. Results obtained have been compared with those computed by empirical relationships available in the literature [21,25,26]. It has been demonstrated that the field thermal probe can be used quite effectively to determine thermal resistivity of various soils and in particular the coarse-grained soils. Further, efforts have been made to develop generalized relationships which can be used for determining the soil thermal resistivity for natural soil deposits, depending upon their dry density, moisture content and different size fractions, in particular gravels, present in the soil.

2. A brief description of the "transient method"

For rapid measurement of the soil thermal resistivity, transient method has been employed by many researchers [17]. It has been noticed that the method is quite convenient and to be adopted for accurate measurement. The method is based on the fact that the rate of temperature of heated body depends upon the thermal coefficient of the material in which it is buried. The basic assumption in this method is that the heating element is straight line of infinite length and infinitely small diameter. And it is embedded in a homogeneous and isotropic medium of infinite extent.

3. Working principle of the "thermal probe"

A thermal probe approximates a line source of heat input of Q per unit length, of constant strength, in an infinite homogeneous soil medium maintained, initially at uniform temperature. Temperature at any point in the soil medium depends on the duration of heating and the soil thermal conductivity. In the mathematical form this can be presented as [17]:

$$\frac{\partial \theta}{\partial t} = \alpha \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \theta}{\partial r} \right) \tag{1}$$

where θ is temperature of the soil mass, t is the time of heating, α is the thermal diffusivity constant ($= k/\gamma C_p$), k corresponds to the thermal conductivity (inverse of thermal resistivity R) of the soil, C_p is the specific heat of the soil, γ is unit weight of the soil, and r is the radial distance from the heat source.

The temperature rise $\Delta\theta$, between the times t_1 and t_2 may be represented as:

$$\Delta\theta = \frac{Q}{4\pi k} \log_e \left(\frac{t_2}{t_1} \right) \tag{2}$$

As such, a plot of temperature against logarithm of time shows a straight portion of slope $\boldsymbol{s}$ that can be defined as:

$$\boldsymbol{s} = \frac{Q}{4\pi k} = \frac{QR}{4\pi} \tag{3}$$

and

$$Q = i^2 \xi \tag{4}$$

where ξ is the resistance per unit length of the probe and i is the electrical current. It will be noted that the average property of the body entering this expression is the thermal conductivity, and that the other terms in Eq. (3) are quantities readily measurable by the developed probes. Eq. (3) can be written as:

$$R = \left(\frac{4\pi}{2.303 i^2 \xi} \right) \times \boldsymbol{s} \tag{5}$$

3.1. Fabrication details of the probe

A laboratory thermal probe developed by Rao and Singh [20] is depicted in Fig. 1. This probe consists of an insulated nichrome heater wire ($\xi = 19.23\ \Omega \cdot m^{-1}$) inserted in a copper tube of 140 mm length with external diameter of 2.5 mm, respectively. The nichrome heater wire is used

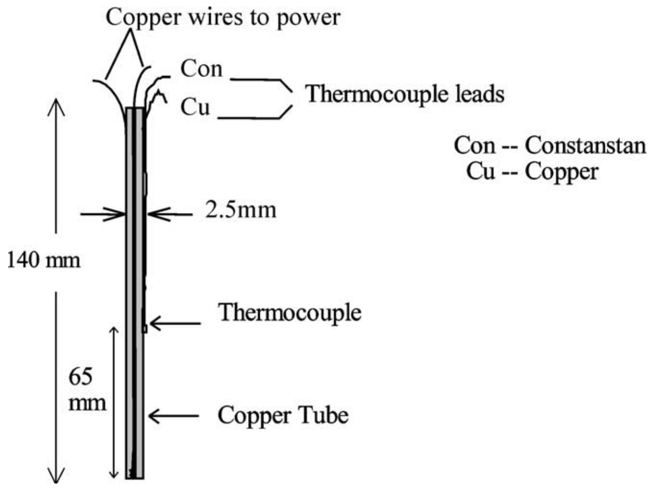

Fig. 1. The laboratory thermal probe.

for heating the probe by passing current through it. MgO of very low resistance (100×10^{-6} to 200×10^{-6} Ω) is used as a filler material so as to insure uniform dissipation of the heat generated. A thermocouple is attached on the surface of the tube to measure the temperature of the probe. However, as this probe could not be used for measuring thermal resistivity of gravels and mixtures of gravels due to small sizes of the probe and the container used for making the test soil sample, a field thermal probe, as shown in Fig. 2, has been fabricated. This probe also works on the principle of the transient method and is an enlarged version of the laboratory probe.

As depicted in Fig. 2, the field thermal probe consists of a hollow copper tube of 1000 mm length with external and internal diameters of 10.5 mm and 7.0 mm, respectively. Nichrome heater wire is used for heating the probe by passing current through it. MgO of very low resistance (100×10^{-6} to 200×10^{-6} Ω) is used as a filler material so as to insure uniform dissipation of the heat generated. ξ of the nichrome wire used for the field thermal probe is 4 $\Omega \cdot m^{-1}$. Three thermocouples: PT100-1, PT100-2 and PT100-3 are provided, on the inner surface of the tube, at distances of 50 mm, 500 mm and 950 mm, respectively, from the bottom of the probe, to measure temperature of the probe. As depicted in Fig. 2, a removable mild steel casing is provided on the top of the filed probe. This casing provides space for housing the circuitry and serves as a sitting place for a mild steel cap. By tamping this cap, the probe can be inserted in the soil mass. However, for very stiff soil deposits a stainless steel dummy rod (1000 mm in length and 9.5 mm in diameter) must be used first for making a hole in the soil deposit followed by insertion of the probe. This insures that there is no air gap between the probe and the surrounding soil.

3.2. Calibration of the probe

To demonstrate proper and efficient functioning of the probe standard glycerol has been used as the medium. The properties of the glycerol used in the study are presented in Table 1 [20].

Table 1
Properties of the glycerol used in the present study

Property	Value
Weight (g) per ml., at 20 °C	1.255–1.260
Neutrality	A 20% solution is neutral to litmus
Maximum limits of impurities: Ash	0.02%
Thermal resistivity value	3.49 $m \cdot K \cdot W^{-1}$

A 1200 mm long and 100 mm diameter glass tube was used for calibration of the probe. The glass tube was filled with the glycerol and the probe was allowed to gain thermal equilibrium with the glycerol. The controlled power supply was switched on followed by recording of the probe temperature (corresponding to three PT100s) as a function of time, till no appreciable change in temperature is noticed. A typical data sheet used for recording these observations is presented in Table 2 for the sake of completeness. Using the recorded data, temperature vs. time and temperature vs. log(time) relationships were developed, as shown in Figs. 3(a) and 3(b), respectively. The value of s for the probe is found to be equal to 30.49 °C, 30.49 °C, and 30.23 °C for thermocouples PT100-1, PT100-2 and PT100-3, respectively. Hence, the thermal resistivity of the glycerol (obtained from Eq. (4), using the field thermal probes is found to be equal to 3.38 $m \cdot K \cdot W^{-1}$. An average s value of 30.40 °C has been used for the field thermal probe. It can be seen that the computed thermal resistivity value is within 3% of the accepted value for glycerol. This indicates that the fabricated ‘field thermal probe’ works very efficiently and hence this probe has been used for estimating thermal resistivity of various soils.

4. Measurement of soil thermal resistivity using the “laboratory thermal probe”

Experiments were carried out to measure the thermal resistivity values of different soils, i.e., clay (black cotton soil), silt (fly ash), silty sand, fine sand, and coarse sand at different densities and moisture contents [20]. Physical properties of these soils are presented in Table 3 and their particle size distribution characteristics are depicted in Fig. 4. A metal container (150 mm long and 100 mm diameter) was used to prepare the samples of soils corresponding to a particular dry density. A 2 mm-diameter hole was drilled in the soil sample and the thermal probe was fit tightly into it. The probe was allowed to achieve thermal equilibrium in the soil mass and then the power supply to the probe was switched on. The temperature of the probe was recorded as a function of time and was used to compute the thermal resistivity of the soil as explained in the calibration section. Based on these results generalized thermal resistivity equations, termed as DDTHERM, have been proposed by Singh and Devid [21]. Validity of these

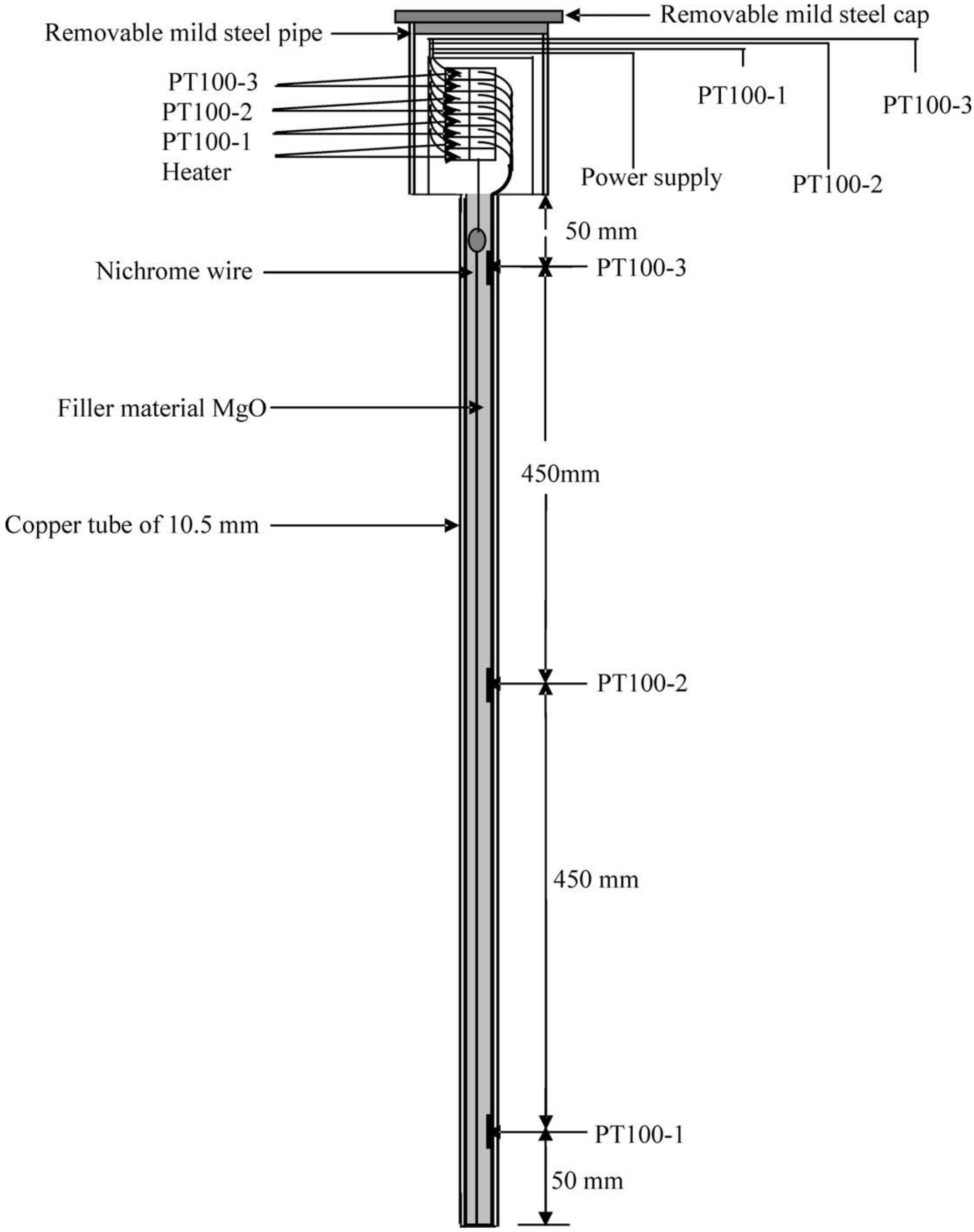

Fig. 2. The field thermal probe.

equations has been done with the help of results available in the literature [27].

5. Measurement of soil thermal resistivity using the "field thermal probe"

For testing thermal resistivity of the soil with the help of field thermal probe, the soil was compacted in a cylindrical PVC container (1105 mm long and 240 mm in inner diameter). By varying the number of layers and number of blows, the required dry density of the soil mass can be achieved. With the help of a dummy steel rod (10 mm in diameter) which is slightly less than the diameter of the probe (=10.5 mm) a hole is drilled along the longitudinal axis of the soil sample, followed by the insertion of the thermal probe. This arrangement insures a proper contact between the probe and the surrounding soil. Later, the power supply is switched on and with the help of temperature readout unit, temperature of three thermocouples is measured as a function of time [28].

This procedure was adopted for determining thermal resistivity of gravels (G), and commercially available standard

Table 2
Data sheet used for the field thermal probe: Material: Glycerol, working density ($kN{\cdot}m^{-3}$): –, room temperature (°C): 27.5, specific gravity: 1.255, moisture content (%): –, current: 3.5 A

Time (min)	Temperature (°C)			Time (min)	Temperature (°C)		
	PT100-1	PT100-2	PT100-3		PT100-1	PT100-2	PT100-3
0	27.2	27.6	27	8.5	62.9	63.1	61.7
0.25	32.6	32.7	32.6	8.75	63	63.3	61.8
0.5	35.2	35	35.1	9	63.4	63.6	62.1
0.75	37.8	37.6	37.8	9.25	63.7	64	62.4
1	39.6	39.4	39.7	9.5	64.2	64.4	62.7
1.25	41.5	41.3	41.4	9.75	64.4	64.6	63
1.5	43.2	43.1	43.4	10	64.7	64.8	63.2
1.75	44.6	44.5	44.7	10.25	65.2	65.3	63.5
2	45.9	45.8	46	10.5	65.5	65.6	63.7
2.25	47.2	47.1	47.2	10.75	65.8	65.9	64
2.5	48.2	48.1	48.2	11	66	66.1	64.1
2.75	49.2	49.2	49.2	11.25	66.4	66.5	64.5
3	50.3	50.2	50	11.5	66.7	66.8	64.7
3.25	51.2	51.2	50.9	11.75	67	67.1	64.9
3.5	52.2	52.1	52.2	12	67.1	67.3	65.2
3.75	52.9	52.8	52.9	12.25	67.6	67.7	65.5
4	53.8	53.8	53.7	12.5	67.7	67.9	65.7
4.25	54.6	54.6	54.4	12.75	68	68.2	65.9
4.5	55.4	55.5	54.5	13	68.1	68.3	66.1
4.75	55.8	55.8	55.3	13.25	68.3	68.5	66.2
5	56.3	56.3	56.2	13.5	68.5	68.7	66.4
5.25	56.9	57	56.7	13.75	68.8	68.9	66.6
5.5	57.5	57.6	57.1	14	69	69.1	66.8
5.75	58	58.1	57.6	14.25	69.3	69.4	67
6	58.5	58.6	58	14.5	69.4	69.6	67.1
6.25	59.1	59.2	58.5	14.75	69.6	69.8	67.3
6.5	59.6	59.6	59	15	69.9	70	67.4
6.75	60.1	60.2	59.3	15.25	70.2	70.4	67.7
7	60.5	60.6	59.6	15.5	70.5	70.6	68
7.25	61.1	61.2	60.1	15.75	70.7	70.7	68.1
7.5	61.3	61.4	60.2	16	70.7	70.8	68.2
7.75	61.6	61.7	60.5	16.25	71	71.1	68.4
8	61.9	62	60.7	16.5	71.1	71.2	68.5
8.25	62.4	62.5	61.3	16.75	71.2	71.3	68.9

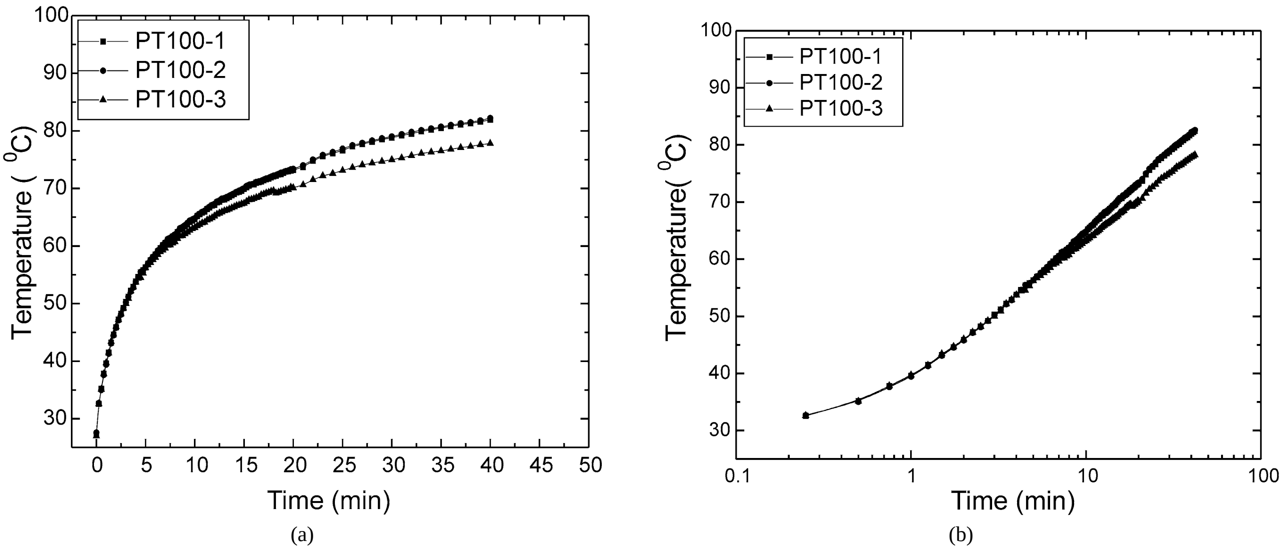

Fig. 3. Variation of the probe temperature with time (for different thermocouples).

Table 3
Soil properties of various soils

Soil type	Notation	G_s	L.L. (%)	P.L. (%)	e_{max}	e_{min}
Clay (Black cotton soil)	BCS	2.72	67	34	–	–
Silt (Fly ash)	FA	2.14	–	–	–	–
Silty sand	SS	2.78	41	28	–	–
Fine sand	FS	2.65	–	–	0.78	0.54
Coarse sand	CS	2.63	–	–	0.76	0.62
Gravel	G	2.74	–	–	0.85	0.67
Standard sand						
Grade I	S-I				0.86	0.58
Grade II	S-II	2.65	–	–	0.87	0.63
Grade III	S-III				0.84	0.56
30% Standard sand + 70% Gravel	SG-I					
	SG-II	2.71	–	–	0.70	0.35
	SG-III					
30% Gravel + 70% Black cotton soil	BCG	2.64	–	–	–	–
30% Gravel + 70% Silty soil	SSG	2.64	–	–	–	–

– not applicable.

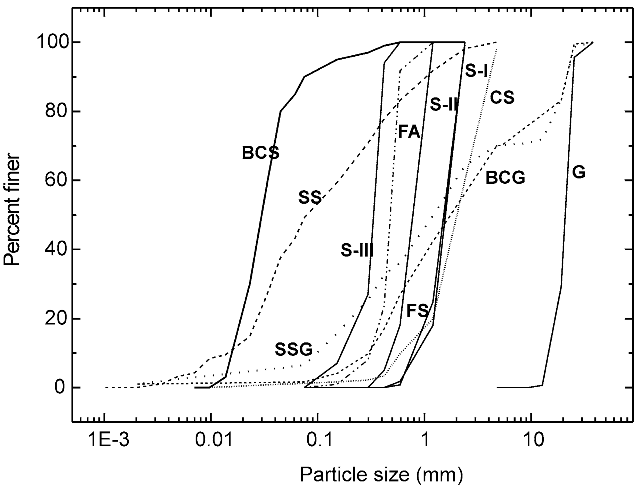

Fig. 4. Particle size distribution characteristics of various soils used in the present study.

sands of various grades Grade I (S-I), Grade-II (S-II) and Grade-III (S-III). Particle size distribution characteristics of these soils are presented in Fig. 4. For the sake of completeness, physical properties of these soils are presented in Table 3. To demonstrate proper functioning of the probe under in situ conditions, where all size fractions (such as gravel, fine and coarse sand, silt and clay etc.) of soils exist, various soil mixtures such as 30% sand and 70% gravel (SG), 70% black cotton soil with 30% gravel (BCG), and 70% silty soil with 30% gravel (SSG) were also tested. Particle size distribution characteristics of these soils, silty soil (SS) and Black cotton soil (BC) are also presented in Fig. 4 and their physical properties are presented in Table 3. The temperature vs. log (time) response of these soils was observed to be similar to the response depicted in Fig. 3. Hence, for the sake of brevity and to avoid repetition, response of these soils is not being presented. The obtained resistivity values for standard sands have been compared with the values obtained from the DDTHERM [21], as shown in Table 4.

Table 4
Comparison of experimental results with DDTHERM

Soil type	γ_{dry} ($kN \cdot m^{-3}$)	R ($m \cdot K \cdot W^{-1}$) Present study	R ($m \cdot K \cdot W^{-1}$) DDTHERM	Difference (%)
S-I	14.5	2.56	2.59	1.17
	15.5	2.34	2.24	4.19
	16.0	2.12	2.08	1.88
S-II	14.4	3.17	3.17	0.11
	15.4	2.78	2.75	1.39
	15.8	2.51	2.59	3.35
S-III	14.6	3.34	3.61	7.37
	15.7	3.06	3.08	0.54
	16.6	2.78	2.71	2.91
G	14.8	5.62	–	–
	16.0	5.07	–	–
	16.4	4.57	–	–
SG I	2.12	2.12	–	–
II	1.95	1.95	–	–
III	1.95	1.95	–	–
BCG	14.0	3.79	–	–
	15.0	3.06	–	–
	15.8	2.78	–	–
	16.5	2.39	–	–
SSG	13.7	3.90	–	–
	15.3	3.01	–	–
	15.7	2.90	–	–
	16.5	2.67	–	–

– could not be obtained.

However, as the laboratory thermal probe could not be used for estimating thermal resistivity of the gravely soils, the obtained thermal resistivity values for gravels could not be compared. This calls for modifying DDTHERM to incorporate effect of gravels on thermal resistivity of a soil with gravel fraction present in it. This has been achieved by following the methodology presented in the following sections.

From the data presented in Table 4, it can be noticed that for standard sands the maximum difference between the

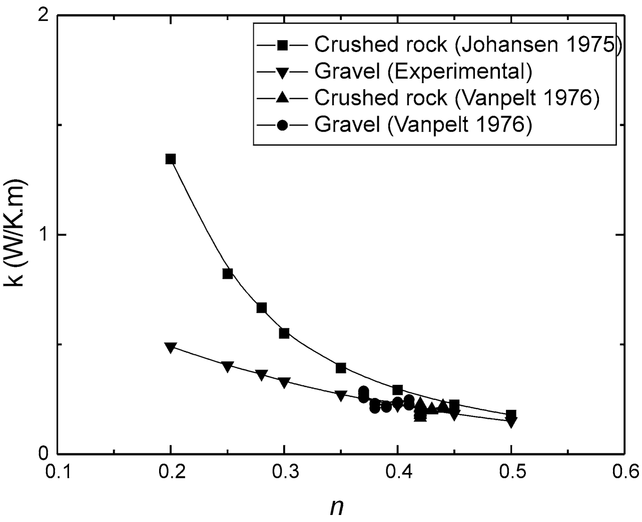

Fig. 5. Comparison of experimental results for gravels with the results reported in the literature.

experimental and the values obtained from the DDTHERM is approximately 0.1 to 7.4%. This indicates that the field thermal probe works quite efficiently and is reliable. It can also be noticed from the data presented in the table that as dry density increases, the value of R for a particular soil decreases. This is due to the improved contact between the soil grains that leads to better conduction of heat [20,29,30]. It can further be noticed that for the finest sand (S-III) the value of R is quite high, for almost comparable dry density, as compared to the coarsest sand (S-I). This is in accordance with the fact that R increases with decrease in the particle size [20,29]. Contrary to this, for gravels (particle size higher than sands), R is quite high as compared to sands. This may be attributed to the fact that for the same value of dry density gravels would yield higher void ratio due to a high specific gravity.

In order to generate more confidence in the proper and efficient functioning of the field probe for its application in determining thermal resistivity of natural soils (i.e., soils with various range of particle sizes), the obtained thermal resistivity values, R, have been converted into the thermal conductivity values, k ($=R^{-1}$), and the same are compared with the results obtained by using Eq. (6) [25] and the experimental results reported by Vanpelt [26], for the crushed rocks and gravels, as depicted in Fig. 5.

$$k = 0.039n^{-2.2} \pm 25\% \tag{6}$$

where k is the thermal conductivity of crushed rocks (in $W \cdot K^{-1} \cdot m^{-1}$) and n is the porosity of the soil (in fraction).

However, as the material for which Eq. (6) can be used is not explicitly mentioned, in the literature [25], it has been assumed that the response of the crushed rocks is similar to gravels to an applied thermal field. It can be noticed from the trends depicted in Fig. 5, that in general, k decreases as porosity increases and the experimental results, for gravels, match very well with the results reported in the literature [25,26] for the crushed rocks and gravels, respectively. This shows that the field thermal probe can be used for measuring thermal resistivity of soils with very large particle size. Another interesting information which can be obtained from Fig. 5 is that porosity of crushed rocks and gravels plays a very critical role in defining its thermal conductivity and its influence is almost negligible for porosity >0.4.

Table 5
Value of ‘a’ for various soils

Soil type	*a*
Clays	0.219
Silts	
Silty-sand	0.385
Fine sand	0.340
Coarse sand	0.480
Gravel	0.21

6. Proposed generalized relationships for estimating soil thermal resistivity

Based on the experimental results obtained by using fabricated field thermal probe, the following generalized relationships have been developed for estimating the soil thermal resistivity.

6.1. Dry (single-phase) soils

For dry soils (single-phase) the following relationship to estimate soil resistivity is being proposed:

$$\frac{1}{R} = 0.01 \times \left[\boldsymbol{a} \cdot 10^{-3+0.06243\gamma_{\text{dry}}}\right] \tag{7}$$

6.2. Moist (single-phase) soils

6.2.1. Clays and silts

To obtain resistivity of moist clays and silts (single-phase) the following relationships are being proposed:

$$\frac{1}{R} = 0.01 \times \left[\boldsymbol{b} \cdot 10^{-3+0.06243\gamma_{\text{dry}}}\right] \tag{8}$$

$$\frac{1}{R} = 0.01 \times \left[1.07\log(w) + \boldsymbol{c}\right] \times 10^{-3+0.06243\gamma_{\text{dry}}} \tag{9}$$

Where, R is the soil thermal resistivity ($m \cdot K \cdot W^{-1}$), w is the moisture content (%) and γ_{dry} is the dry-density of the soil ($kN \cdot m^{-3}$). Parameters $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ depend on the type of the soil and its moisture content and their values are presented in Tables 5, 6 and 7, respectively.

6.2.2. Silts, sands and gravel

Eq. (9) can also be used to predict resistivity of silts and sands.

In order to facilitate computation of thermal resistivity of a multi-phase soil system, generalized relationships have been developed, assuming that soil consists of six-phase system (clays, silts, silty-sand, fine-sand, coarse-sand and

Table 6
Value of 'b' for clays and silts

w (%)	Soil type	b
$4 > w \geqslant 2$	Clays	0.243
	Silts	0.254
$5 \geqslant w > 4$	Clays	0.276
	Silts	0.302

Table 7
Value of 'c' for various soils

Soil type	c	w (%)
Clays	−0.73	>5
Silt (Fly ash)	−0.54	
Silty sand	0.12	⩾1
Fine sand	0.70	
Coarse sand	0.73	
Gravel	0.8	

gravel). For a naturally occurring soil, the resistivity of different phases is calculated by using Eqs. (7)–(9). These resistivity values are multiplied by certain weights, which can be computed on the basis of their phase fraction. The weights assigned to different single-phase soils can be obtained as follows:

6.3. Weights

For clay and silt phase

$$\text{Weight} = (\text{phase }\%), \text{ when } 5 \geqslant w\ (\%) \geqslant 2 \tag{10}$$

$$\text{Weight} = \text{Minimum of the (Absolute } \boldsymbol{c} \text{ value or phase }\%), \text{ when } w\ (\%) > 5 \tag{11}$$

Silty-sand, fine-sand coarse-sand and gravel

$$\text{Weight} = (\text{phase }\% \times \boldsymbol{c} \text{ of the phase}) + \text{phase }\%, \text{ when } w\ (\%) > 1 \tag{12}$$

$$\text{Weight} = \boldsymbol{a} \text{ of the phase, when } w\ (\%) < 1 \text{ (dry soils)} \tag{13}$$

However, if a certain phase is absent, the weight for the phase is assigned as zero. Sum of the resistivity values, so obtained, yields the thermal resistivity of the naturally occurring soil (or a soil mix).

7. Validation of the proposed generalized equations

For establishing efficiency of the proposed generalized equations, and termed as MDDTHERM, the experimental results reported in the literature [27] have been used, as depicted in Table 8. From the table it can be noticed that the proposed equations predict resistivity values, which are very close to the experimental results particularly when the test is conducted for the dry state of the soils. However it must be noticed that the clay fraction (<0.005 mm), as specified in the literature [27], which is greater than 0.005 mm has been considered as the silt. This highlights efficiency of the proposed equations in estimating soil thermal resistivity.

Further validation of MDDTHERM has been conducted with the help of experimental results obtained for the two-phase soils (i.e., SG-I, SG-II, SG-III, BCG and SSG). As depicted in Table 9, the % difference between these resistivity values is only 4 to 13.5%. This indicates that the MDDTHERM works very efficiently and can be employed for estimation of thermal resistivity values of natural soils, knowing the percentage fractions of various particles and its moisture content and dry density.

8. Concluding remarks

Based on the results and discussions presented in the previous sections of this paper, it can be concluded that

Table 8
Validation of the proposed generalized relationships (MDDTHERM)

Soil	Gravel (%)	Coarse sand (%)	Fine sand (%)	Silt (%)	Clay (%)	γ_{dry} (kN·m^{-3})	OMC (%)	R (m·K·W^{-1}) William (1960) OMC		R MDDTHERM OMC	
								OMC	DRY	OMC	DRY
A	–	39.2	47.3	7.0	6.5	18.5	13.3	0.41	1.94	0.49	1.99
B	–	36.8	48.3	7.0	7.5	17.5	9.3	0.53	2.34	0.71	2.33
C	–	36.9	58.1	7.4	7.6	19.7	9.7	0.38	1.55	0.51	1.74
D	–	38.0	46.5	9.0	6.5	17.8	14.0	0.45	2.20	0.54	2.23
E	–	27.10	62.4	5.5	5.0	16.2	16.1	0.54	2.90	0.59	2.80
F	–	13.9	71.6	7.0	7.5	19.5	8.8	0.40	1.63	0.55	1.85
G	–	13.5	70.0	8.5	8.0	17.4	9.8	0.52	2.35	0.72	2.53
H	–	10.9	73.1	8.5	7.5	15.7	10.0	0.66	3.32	0.90	3.26
I	–	28.5	62.0	5.0	4.5	17.1	11.7	0.51	2.46	0.58	2.43
J	12.5	31.5	41.5	8.5	6.0	17.8	10.3	0.49	2.20	0.62	2.44
K	5.8	80.5	11.7	1.2	0.8	18.3	7.5	0.50	2.01	0.48	1.73
L	7.6	46.4	37.0	3.0	6.0	19.3	9.7	0.40	1.68	0.47	1.78
M	17.0	32.0	28.0	15.5	7.5	19.8	10.5	0.36	1.51	0.53	1.94
N	28.6	24.4	32.4	10.6	4.0	18.8	13.3	0.39	1.83	0.46	2.34

– not present.

Table 9
Comparison experimental results with MDDTHERM

Soil type		Dry density ($kN \cdot m^{-3}$)	R ($m \cdot K \cdot W^{-1}$) Experimental	MDDTHERM	Difference (%)
SG	I	19.9	2.12	2.21	4.1
	II	20.2	1.95	2.12	7.9
	III	21.0	1.95	2.03	4.0
BCG		14.0	3.79	3.96	4.4
		15.0	3.06	3.44	10.9
		15.8	2.78	3.06	9.1
		16.5	2.39	2.77	13.5
SSG		13.7	3.90	4.34	10.1
		15.3	3.01	3.45	12.8
		15.7	2.90	3.25	11.0
		16.5	2.67	2.98	10.5

the 'field thermal probe' works quite efficiently. It has been demonstrated that the thermal resistivity of sand and gravels, obtained from this probe, match very well with the results reported in the literature for sands, gravels and the crushed rocks. The generalized relationships for determining thermal resistivity of various soils have been developed and it is observed that these relationships are quite efficient in predicting thermal resistivity of various soils.

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